Ratio – a comparison of two numbers or quantities **with the same units.**

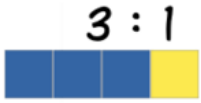


Figure 1: There are 3 black squares to 1 grey square

Ratios can be show in different ways:

a. 2 cups of milk to 7 cups of water 2:7 or $\frac{2}{7}$	**b.** \$5 to \$9 $\frac{5}{9}$ or 5:9

Example 2: Write each ratio in simplest form.

a. $\frac{6}{15}$ GCF = 3 $= \frac{6\div3}{15\div3} = \frac{2}{5}$ or 2:5	**b.** 4:12 $\frac{4}{12}$ GCF: 4 = $\frac{1}{3}$	**c.** 6 to 10 $\frac{6}{10}$ GCF: 2 $\frac{3}{5}$

Example 3: Write the following ratios in simplest form:

a. 45 minutes to 1 hour $\frac{45}{60}$ GCF: 15 60min $= \frac{45\div15}{60\div15} = \frac{3}{4}$	**b.** 250 g to 1kg 1000g $\frac{250}{1000} = \frac{1}{4}$	**c.** 1m to 175cm 100 $\frac{100}{175}$ GCF: 25 $\frac{4}{7}$

Rate – a comparison of two numbers having **different units.**

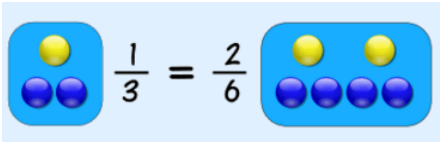
A rate is usually written as a 'unit rate', in which the second term is always 1.

min wage \$11.25/h unit price

speed

Example 4: John earns \$60 for working 4 hours. What is his rate of pay? $\frac{\$60}{4hr} = \$15/h$	**Example 5:** A car runs at a speed of 30m/s. How far can it run in 1 minute? ×60 $\frac{30m}{1s} = \frac{d}{60s}$ ×60 $d = 30 \times 60$ $d = 1800m$
Example 6: A 200g bag of mixed nuts costs \$3.40. Calculate the unit rate. (price) $\frac{\$3.40}{200g} = \$0.017/g$	**Example 7: A Comparison** A 200g bag of popcorn costs \$6.00. A 500g bag costs \$10.00. Find the unit rate of each bag to compare which size is the better value. OPTION A $\frac{\$6.00}{200g} = \$0.03/g$ OPTION B $\frac{\$10.00}{500g} = \$0.02/g$ ∴ Option B is cheaper

Proportion – is an equation which states that two ratios are equal. $\frac{a}{b} = \frac{c}{d}$



Some proportions can be solved with simple multiplication or division between equivalent ratios, others are more complicated and can be solved using 'cross multiplication'.

Example 8: $\frac{3}{4} = \frac{x}{12}$ ×3, ×3 $3 \times 3 = x$ $9 = x$	**Example 9:** $\frac{5}{x} = \frac{25}{15}$ ÷5 $x = 15 \div 5$ $x = 3$	**Example 10:** $\frac{1.4}{x} = \frac{30}{25}$ *see below $1.4 \cdot 25 = 30 \cdot x$ $\frac{35}{30} = \frac{30x}{30}$ $x \doteq 1.16$

Cross multiplication

Example 1: Find the missing value 'm'

*You should be able to answer this by solving the 'simple' equivalent fraction but I will use this simple example to show you how cross multiplication works.

Question	Draw the cross	STEP 1: Set up the equation	STEP 2: Simplify	STEP 3: Get the unknown value alone by dividing both sides by the number on the same side as the unknown value.
$\frac{1}{5} = \frac{m}{8}$	$\frac{1}{5} \times \frac{m}{8}$	$1 \times 8 = m \times 5$	$8 = m \times 5$	$\frac{8}{5} = \frac{m \times 5}{5}$ $1.6 = m$

Example 2:

Question	Draw the cross	STEP 1: Set up the equation	STEP 2: Simplify	STEP 3: Get the unknown value alone by dividing both sides by the number on the same side as the unknown value.
$\frac{4.5}{6} = \frac{3.6}{m}$	$\frac{4.5}{6} \times \frac{3.6}{m}$	$4.5m = 3.6 \cdot 6$	$4.5m = 21.6$	$\frac{4.5m}{4.5} = \frac{21.6}{4.5}$ $m = 4.8$

Proportion Problems:

Example 11:
A pendulum completes 7 swings every three seconds. How many swings does it complete in a minute? 1 min = 60

$\frac{7}{3} \times \frac{x}{60}$

$7 \cdot 60 = 3x$

$\frac{420}{3} = \frac{3x}{3}$

$140 = x$

∴ I'll complete 140 swings

Example 12: Apples are $2.00 per dozen (12), how many apples can you get for $5.50?

$\frac{2}{12} = \frac{5.5}{x}$

$2x = 12 \cdot 5.5$

$\frac{2x}{2} = \frac{66}{2}$

$x = 33$

∴ You get 33 apples

Example 13: A 17" computer monitor has a width of 15". Since monitors are proportionate, what is the width of 48" computer monitor?

$\frac{17}{15} = \frac{48}{x}$ ← width

$17x = 15 \cdot 48$

$\frac{17x}{17} = \frac{420}{17}$

$x \doteq 24.7$

∴ Its width is 24.7"

Practice: Ratios, Rates, and Proportions

Write the following as ratios in lowest terms

a. 73 days to 1 year

$\frac{73}{365} = \frac{1}{5}$ or 1:5

b. 35cents to $1.05

$\frac{35}{105} = \frac{1}{3}$ or 1:3

c. 750 mL to 1.5 L

$\frac{750}{1500} = \frac{1}{2}$ or 1:2

d. 3 min to 45 sec

$\frac{180}{45} = \frac{4}{1}$ or 4:1

Find the unit rate of the following:

e. Mike earns $42 in 6 hours.

$\frac{\$42}{6h} = \$7/h$

f. $350 for 8 people to attend a party

$\frac{\$350}{8 \text{ people}} = \$43.75/\text{person}$

g. 24 pop for $6.96

$\frac{\$6.96}{24 \text{ can}} = \$0.29/\text{can}$

Answer the following rate problems:

h. Jack earned $50 in 10 hours, while John earned $105 in 20 hours. Which person had the better rate of pay?

Jack: $\frac{\$50}{10h} = \$5/h$

John: $\frac{\$105}{20h} = \$5.25/h$

∴ John has the better rate of pay.

i. A bus travels 10 km in 25 minutes. At this rate, how far will the bus travel in one hour?

$\frac{10km}{25min} = \frac{d}{60min}$

$10 \cdot 60 = 25d$

$\frac{600}{25} = \frac{25d}{25}$

$d = 24$

∴ It'll travel 24km in one hour.

j. Oranges are $2.00 per dozen. At this rate, how many oranges could you get for $3.50

$\frac{\$2}{12} = \frac{\$3.5}{x}$

$2x = 12 \cdot 3.5$

$\frac{2x}{2} = \frac{42}{2}$

$x = 21$

∴ You'll get 21 oranges

k. Katherine cycled 30 km in 2 hours. If she continues at the same rate, what distance will she travel in 7 hours?

$\frac{30km}{2h} = \frac{x}{7h}$

$30 \cdot 7 = 2x$

$\frac{210}{2} = \frac{2x}{2}$

$x = 105$

∴ She'll cycle 105km in 7 hours.

l. Which is the better value? $350 for a bus of 35 people, or $440 for a bus of 40 people?

A: $\frac{\$350}{35} = \$10/\text{person}$

B: $\frac{440}{40} = \$11/\text{person}$

∴ Option A is the better rate by $1

m. Which is the better value? 28 g of mixed nuts for $0.84, or 35g of mixed nuts for $1.40?

A: $\frac{0.84}{28} = \$0.03/g$

B: $\frac{1.4}{35} = \$0.04/g$

∴ Option A is the better value by 1 cent

Find the missing value in the following proportions *round to 2d.p. where necessary

n. $\frac{3}{8} = \frac{m}{5}$

$3.5 = 8m$

$\frac{3.5}{8} = \frac{8m}{8}$

$1.88 = m$

o. $\frac{2}{k} = \frac{11}{4.5}$

$2 \cdot 4.5 = 11k$

$\frac{9}{11} = \frac{11k}{11}$

$k = 0.82$

p. $\frac{1.2}{2.8} = \frac{3}{p}$

$1.2p = 2.8 \cdot 3$

$\frac{1.2p}{1.2} = \frac{8.4}{1.2}$

$p = 7$

q. $\frac{5}{3.2} = \frac{2.5}{y}$

$5y = 2.5 \cdot 3.2$

$\frac{5y}{5} = \frac{8}{5}$

$y = 1.6$

ANSWERS: a. 1:5, b. 1:3, c. 5:6, d. 4:1, e. $7/h, f. $43.75/p, g. $0.29/pop, h. John, i. 24km/h, j. 21oran., k. 105 km, l. $350/35, m. 28g/$0.84, n. m=1.88, o. k=0.82, p. p=7, q. y=1.6